μ-MRI Simulation Tool Development: An MRI Simulation Tool Software Based on Bloch’s Equation for Studying
the Magnetic Computing and Pulse Sequencing Research in Magnetic Resonance Imaging

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Abstract— Magnetic resonance imaging (MRI) currently becomes an essential radiological imaging tool because of its exquisite image resolution, versatility applications, and non-ionizing radiation. Although MRI scanners are available for more than twenty years, the system and maintenance costs are prohibited, preventing its uses for general patients in Thailand and for MRI research. For this reason, understanding the science behind the scanners and researching in this field are limited. This work aims to develop simulation software that simulates MR signals from a phantom after applying a specific pulse sequence and to reconstruct images from these MRI signals. This simulation is based on the solution of 3D discrete Bloch’s equation. The simulation can be divided into three parts: phantom definition, magnetization kernel simulation, and image reconstruction. The software is implemented on MATLAB.

Keywords-component: Bloch’s equation, magnetization kernel, magnetic resonance imaging, pulse sequence, simulation

1. INTRODUCTION

Medical imaging system is an important tool to realize and create human body image for clinical purposes. Each modalities of medical image has its own features such as soft tissue image, structural image and functional image. Medical images are used in clinical for diagnosis an abnormal in human body, for planning in an operation, and for the post estimation after surgery procedures to check physical conditions. One of the major tools in radiology is magnetic resonance imaging (MRI). This type of images is used to diagnose the pathologies in all parts of the body with neurology, cardiology, hepatic, nephrology, and musculoskeletal applications. New MRI techniques of MRI enhance angiography, blood perfusion, water perfusion, and localization of functional brain activation.

MRI is a tomographic imaging technique that produces image of internal physical and chemical characteristics by measuring the signals from protons of water and lipid inside human body. This modality is based on the well-known nuclear magnetic resonance (NMR) phenomena [2]. NMR involves with nuclei, magnetic fields, and the resonance phenomenon. Nuclear spin is an important property, called nuclear magnetism that is affected by external magnetic field. Image information is gathered from spatial encoded signals. These signals are uniquely encoded to spatial frequency, and are detected from the outside of the objects. MRI machine uses the static powerful magnetic field, radio frequency (RF) pulse, and the gradient magnetic field. All of these components work to generate and manipulate the signals.

In comparison to other image modalities [7], the main advantages of MRI are: (i) No ionizing radiation is required, (ii) The images can be acquired in any two or three dimension plane, (iii) The excellent of soft-tissue contrast, (iv) A spatial resolution of the order for 1 mm or less can be read, and (v) Images are produced with negligible penetration effects. In contrast, MRI also has some disadvantages: (i) MRI acquisition speed is about the same PET and much slower than CT and ultrasound, e.g., the typical clinical protocol might last 30-40 minutes, (ii) A significant percentages of patients are precluded from MRI scans due to metallic implants from previous surgeries, and (iii) MRI systems are much more expensive than CT or ultrasound units.

The process in generating of MR images can be divided into three main steps. First is to alter the random directions of proton magnetizations by using of strong static magnetic fields to align them along the magnetic field direction. Second is to manipulate the magnetizations in order to generate signals from these magnetizations. Finally, these signals are collected using RF coils and reconstructed using Fourier transform to be MR images.

MR images obtained from same anatomy site can illustrate different magnetization properties of the specimens by varying data acquisition protocols using
pulses. The MRI pulse sequence is a sequence of magnetization manipulation events[6]. The event in a pulse sequence consists of RF pulse transmission, gradient field superimposition for phase and frequency encoding, and data acquisition. The events are arranged in a sequence to enhance particular magnetization properties in specimens.

RF pulses are used to systematically alter alignment of the magnetizations to a specific direction. Their frequencies are based on the strength of the main static magnetic field. After removing the RF pulse, the magnetizations gradually dephase due to the inequality of spin frequencies and simultaneously return to their equilibrium states, i.e., parallel to the main magnetic field direction. From this changing, the MR signals can be recorded using RF coils based on Faraday’s law of induction. The MR signals are dependent on tissue physical parameters, including the nuclear spin density (ρ), the spin-lattice relaxation time (T1), the spin-spin relaxation time (T2), molecule motions, susceptibility effects, and chemical shift differences[4].

Gradient magnetic fields are small static magnetic fields that their strength is varying linearly along spatial directions. These gradient fields superimpose on the main magnetic field in order to vary the spin frequencies along the spatial directions. They are used to encode spatial information into the responses of the spins. They are applied in slice-selective, phase, and frequency encoding directions. They are also used to rephase the spins, i.e., undo the dephasing process, in order to recover the signals and make an echo, i.e., when the spins are unwired and aligned in one direction.

Simulation of these MR signals is an economic initiative way to study MR phenomena, to investigate the causes of artifacts and their effects, to develop and optimize MRI pulse sequences for MRI research. In literature, simulation could be divided into four categories. The first category uses only proton density, T1, and T2 to map and simulate the MR signals acquired using different repetitions and echo times. However, this approach is not designed to stimulate the entire MR phenomena and some image artifacts, e.g., chemical shift, intra-voxel and dephasing. The second category uses the k-space formalism[5] to create k-space amplitudes. Each type of tissues must be treated separately to simulate the non-uniform characteristics. The third category is hybrid approach that each tissue type is associated with a spin model that stimulate intra-voxel heterogeneity by replacing different frequency. The fourth category is based on a discrete-event Bloch’s equation[1, 3]. This approach is the closest to the reality.

This work aims to study the MRI process and to develop a simulation software. This simulation is based on the 3D discrete Bloch’s equation. The simulation can be divided into three parts: virtual object definition, magnetization computing simulation, and image reconstruction. The software is implemented on MATLAB. Preliminary results show that the reconstructed images can be generated by the virtual object which is computed with desire pulse sequence, 2D spin-echo pulse sequence, in MRI process.

2. MATERIAL & METHOD

2.1 Methods

This simulation is divided into three parts: virtual object definition, magnetization computing, and spin echo pulse sequence definition (Fig. 1). Virtual object and system configuration are configured, and then these data are computed in magnetization computing. After magnetization computing computed MR signal in MRI process, the MR signal is collected into k-space domain data. Finally, these k-space domain data will be reconstructed, and become the images.

![Figure 1: Simulation flow diagram](image)

A. Virtual Object

Virtual object is divided in voxels in a rectangular grid. This object contains several physical parameters that are essential to compute the local spin magnetization, which are proton density (ρ), longitudinal relaxation time (T1) and transverse relaxation time (T2). The virtual object describes a nuclear spin system of one component with a proton of water. In order to test 2D spin echo pulse sequence, the virtual object in this work is 3D virtual with size of X × Y × Z voxels.

B. Magnetization Computing

Magnetization computing is based on the solution 3D discrete Bloch’s equation. The magnetization vector evolves with time t is $M = [M_x, M_y, M_z]$.

$$\frac{dM(t)}{dt} = \gamma (M(t) \times B) - \frac{M_x(t)}{T_1} - \frac{M_y(t)}{T_2} - \frac{M_z(t)}{T_2}$$

where M(t) is the magnetization vector at time t, B(t) is the magnetic field at time t, T1 and T2 are the relaxation
times, and \( i, j, k \) are the vectors of perpendicular axes \( x, y, z \). The local magnetic field can be defined as

\[
B = B_0 \cdot \vec{i} + G(t) \cdot \vec{r} + B_1(t)
\]

(2)

where \( B_0 \) is static magnetic field, \( G(t) \) is an applied gradient, \( B_1(t) \) is radio frequency magnetic fields and \( r = (x,y,z)^T \) is Cartesian coordinates. The magnetization vector is computed using the following equation:

\[
\vec{M}(\vec{r}, t + \Delta t) = R_z(\theta) \cdot R_{\text{rel}} \cdot \vec{R}_{\text{rf}} \cdot \vec{M}(\vec{r}, t)
\]

(3)

where \( R_z \) is a rotation matrix about \( z \) axis which describes effects of static magnetic fields and gradient magnetic fields, as shown in equation (4).

\[
R_z = \begin{bmatrix}
\cos(\theta) & -\sin(\theta) & 0 \\
\sin(\theta) & \cos(\theta) & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(4)

which \( \theta \) is

\[
\theta = \int_{0}^{\frac{\pi}{2}} \gamma G(t) dt
\]

(5)

For \( R_{\text{rel}} \) describe relaxation effects of spins

\[
R_{\text{rel}} = \begin{bmatrix}
e^{-\frac{M_t}{T_1}} & 0 & 0 & M_s(0) \\
0 & e^{-\frac{M_t}{T_2}} & 0 & M_s(0) \\
0 & 0 & e^{-\frac{M_t}{T_2}} & M_s(0) \\
0 & 0 & 0 & 1
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]

(6)

where \( \vec{R}_{\text{rf}} \) is representing the rotation effect of RF pulse with flip angle \( \theta \), and \( \phi \) is a phase angle leading to flip angle.

\[
\vec{R}_{\text{rf}} = \begin{bmatrix}
\cos(-\phi) & -\sin(-\phi) & 0 \\
\sin(-\phi) & \cos(-\phi) & 0 \\
0 & 0 & 1
\end{bmatrix}
\]

(7)

C. Spin Echoes Pulse Sequence

An echo signal can be generated by multiple RF pulse. The signals of the former type are called RF echoes. To generate an RF echo, there must be at least two pulses. Beginning with a 90 degree pulse, followed by the time delay, and then applies another 180 degree pulse. The generated echo signal is called a spin echo.

A simulation flow diagram as shown in Figure 2, is used for graphical understanding. The event of pulse sequence is separated into several parts. Firstly, while a Sinc RF pulse of 90 degree, i.e., \( \pi/2 \), is generated, the gradient magnetic field is also applied. This step excites the magnetization within the bandwidth of RF. Second, a rephrasing gradient is used to rephase the spins that lose their phase coherence. The area of the rephrasing gradient is equal one-half area of applying slice selective gradient. The next step is the phase encoding. The phase encoding gradient is applied to make magnetization in the virtual object to have the different phases in the direction of \( y \)-axis in this simulation, and then a refocusing RF pulse with 180 degree and slice selective gradient are applied to reverse the phase of magnetization in order to generate an echo signal. For the rectilinear sampling of the \( k \)-space, the pre-readout gradient has to perform before the frequency encoding. The final step of the 3D spin echo pulse sequence is the frequency encoding and the data collection. The frequency encoding and the data acquisition are applied in order to sampling a data point with a different frequency in the \( x \)-direction for this simulation.

3. PRELIMINARY TEST AND ITS RESULTS

A. Virtual object

![Virtual object](image)

Figure 3: A virtual object’s cross-sectional image showing two different tissue types (white and gray blocks) and empty space (black blocks).

A virtual object for this simulation comprises of four rectangular blocks as shown in Fig. 3. Two different tissue types, Tissue A and Tissue B, are represented the white and gray blocks, respectively, while an empty space is shown by the black blocks. The dimension of this virtual object is 10x10x10 voxels. For this
simulation, Tissue A proton density, T1, and T2 parameters are different from Tissue B.

B. Preliminary for function testing

Figure 4: Magnetization evolution in spin echo pulse sequence

Figure 4 is obtained from 1 point of virtual object with using the spin echo technique. Example tissue has time relaxation time of T1 = 2,500 ms and T2 = 600 ms. The sequence parameter of TE = 40 ms and TR = 250 ms for each iteration. For Mx and My are the magnetization in X-axis and Y-axis or transverse axis, and Mz is the magnetization in longitudinal axis or Z-axis.

C. 2D image from 3D virtual object

Figure 5: Virtual object, k-space data, and reconstructed images

The preliminary results shown in Fig. 5 are the virtual object, the k-space data from the proposed simulation, and the MR image reconstructed from this k-space data. The virtual objects used in 2D spin echo have two different tissue types. After pass through all events of the 2D spin echo pulse sequence, k-space complex signals are collected one line for each iteration for ten lines. Each line is composed of ten data points. This k-space data are then Fourier transforms creating an MR image.

4. CONCLUSION

This paper presented an overview of simulation and image reconstruction for MRI. An algorithm that was implemented is based on the solution of 3D discrete Bloch’s equation. This simulation enables to stimulate a 2D image from a virtual 3D object. The virtual object has several physical parameters, such as, proton density, transverse relaxation time and longitudinal relaxation time. There are several configurable parts in MRI process. Firstly, an arrangement of the process includes RF pulse, gradient magnetic field, and data acquisition is settled. Secondly, the simulation can generate RF pulse in the desire shape. For the part, the gradient magnetic fields can be generated the different types in order to apply phase encoding and frequency encoding. This part can be changed for applying with different types of sampling methods. The final part is a data acquisition which can set the acquisition time.

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REFERENCES